Tests of Hypotheses

1. **Test a Population Mean**

   Null Hypothesis: $H_0: \mu = \mu_0$

   Test Statistic: $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a: \mu > \mu_0$	$z > z_\alpha$ (upper-tailed test)
$H_a: \mu < \mu_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a: \mu \neq \mu_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   If we don't have a normal population, the sample size should be large i.e., $n > 30$. If the population standard deviation σ is not known, we use the sample standard deviation s, in this case we need $n > 40$ even if the population is normal.

2. **Test a Population Proportion**

   Null Hypothesis: $H_0: p = p_0$

   Test Statistic: $z = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a: p > p_0$	$z > z_\alpha$ (upper-tailed test)
$H_a: p < p_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a: p \neq p_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   The sample size should be large i.e., $np_0 > 10$ and $nq_0 > 10$.

3. **Test a Difference Between two Population Means**

   Null Hypothesis: $H_0: \mu_1 - \mu_2 = D_0$

   Test Statistic: $z = \dfrac{\bar{x}_1 - \bar{x}_2 - D_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

Alternative Hypothesis	Rejection region at the significance level α
$H_a: \mu_1 - \mu_2 > D_0$	$z > z_\alpha$ (upper-tailed test)
$H_a: \mu_1 - \mu_2 < D_0$	$z < -z_\alpha$ (lower-tailed test)
$H_a: \mu_1 - \mu_2 \neq D_0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   If we don't have normal populations, the sample sizes should be large i.e., $n_1 > 30$ and $n_2 > 30$, and the two samples should be independently randomly selected. If σ_1 and σ_2 are not known, we use s_1 and s_2, in this case we need $n_1 > 40$ and $n_2 > 40$.

4. **Test a Difference Between two Population Proportions**

   Null Hypothesis: $H_0: p_1 - p_2 = 0$

   Test Statistic: $z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}(1/n_1 + 1/n_2)}}$, where $\hat{p} = \dfrac{n_1\hat{p}_1 + n_2\hat{p}_2}{n_1 + n_2}$.

Alternative Hypothesis	Rejection region at the significance level α
$H_a: p_1 - p_2 > 0$	$z > z_\alpha$ (upper-tailed test)
$H_a: p_1 - p_2 < 0$	$z < -z_\alpha$ (lower-tailed test)
$H_a: p_1 - p_2 \neq 0$	$z > z_{\alpha/2}$ or $z < -z_{\alpha/2}$ (two-tailed test)

   The sample sizes should be large i.e., $n_1\hat{p}_1, n_1\hat{q}_1, n_2\hat{p}_2$, and $n_2\hat{q}_2$ should all be greater than 10, and the two samples should be independently randomly selected.

5. ***p*-value**

   The p-value is the smallest value of α for which the null hypothesis could be rejected. It is the probability that the null hypothesis could produce an observed sample at least as extreme as the one that was observed. The smaller the p-value, the stronger the evidence against H_0.

   For an upper-tailed test, the p-value is $P(Z > z_{\text{obs}})$.

   For a lower-tailed test, the p-value is $P(Z < z_{\text{obs}})$.

   For a two-tailed test, the p-value is $P(Z > |z_{\text{obs}}|) + P(Z < -|z_{\text{obs}}|) = 2\,P(Z > |z_{\text{obs}}|)$.

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