

Chi-Square Distribution

Inference about a Population Variance

If the population from which the sample is selected is (approximately) normally distributed, then

$$\frac{(n-1)s^2}{\sigma^2}$$

has a chi-square distribution with $n - 1$ degrees of freedom.

A $(1 - \alpha)$ 100% Confidence Interval for σ^2 is of the form

$$\frac{(n-1)s^2}{\chi_{\alpha/2}^2} < \sigma^2 < \frac{(n-1)s^2}{\chi_{(1-\alpha/2)}^2}.$$

The confidence interval for σ can be obtained by taking the square root of the two limits of the above interval.

Test about a Population Variance

Null Hypothesis: $H_0: \sigma^2 = \sigma_0^2$

Test Statistic: $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

Alternative Hypothesis

$$H_a: \sigma^2 > \sigma_0^2$$

$$H_a: \sigma^2 < \sigma_0^2$$

$$H_a: \sigma^2 \neq \sigma_0^2$$

Rejection region at the significance level α

$$\chi^2 > \chi_{\alpha}^2 \text{ (upper-tailed test)}$$

$$\chi^2 < \chi_{\alpha}^2 \text{ (lower-tailed test)}$$

$$\chi^2 > \chi_{\alpha/2}^2 \text{ or } \chi^2 < \chi_{(1-\alpha/2)}^2 \text{ (two-tailed test)}$$

Goodness-of-fit Test

A **Multinomial Experiment** is an experiment with the following properties.

1. It consists of n identical independent trials.
2. Each trial results in one of k possible outcomes.
3. The probabilities p_i of the possible outcomes remain constant for each trial.

The test statistic for a goodness-of-fit test of a multinomial experiment is

$$\chi^2 = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i}$$

where:

O_i = observed frequency of category i , and E_i = expected frequency of category $i = np_i$.

In a goodness-of-fit test, the degrees of freedom are $df = k - 1$. It is always an upper-tailed test. The number of trials n should be large enough so that $np_i > 5$ for all i .

The p -value is $P(\chi^2 > \chi_{\text{obs}}^2)$.