Birthday Problem

If a group of n people is in a room, what is the probability that at least two of them have the same birthday? Ignore leap years and assume that each day in the year is equally likely as a birthday.

Let's consider the sample space of all ordered lists of n birthdays. One such list assigns a birthday to each of the n people. All simple events in this sample space are equally likely to occur. Consider the event

$$E = \text{"at least two people have the same birthday"}.$$

To compute $P(E)$, we first compute $P(\overline{E})$ and then use $P(\overline{E}) = 1 - P(E)$. The complement of E is the event

$$\overline{E} = \text{"no two people have the same birthday"}.$$

Its probability is given by

$$P(\overline{E}) = \frac{365 \cdot 364 \cdot 363 \cdots (365 - n + 1)}{365^n} = \frac{P(365, n)}{365^n}.$$

Therefore,

$$P(E) = 1 - \frac{P(365, n)}{365^n}.$$

Let's compute this probability for a few values of n.

n	$P(E)$
5	0.0271
10	0.1169
20	0.4114
22	0.4757

n	$P(E)$
23	0.5073
30	0.7063
40	0.8912
50	0.9704

Observe that when $n = 23$, the probability exceeds $1/2$ for the first time. This means that if there are 23 or more people in a room, there is more than 1 chance out of 2 that at least two people have the same birthday!

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It is important that we do not confuse the above birthday problem with the following one.

If a group of n people is in a room, what is the probability that at least one person in the group has the same birthday as you.

In this case, let

$$E = \text{"at least one person has the same birthday as you"}.$$

We obtain in this case

$$P(E) = 1 - \left(\frac{364}{365}\right)^n.$$

The smallest value of n where $P(E) > 1/2$ is now $n = 253$ which is much larger than 23.

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